ASSESSMENT OF WEAPON SELECTION FOR ACHIEVING GUARANTEED VICTORY IN TACTICAL ACTION

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The paper deals with the problem of securing a guaranteed victory for our side in the course of military operations, when the dynamic power of the enemy is known. The problem of the set of weapons for a guaranteed victory is solved. A method is given for calculating the set of weapons and equipment that guarantees victory without regard to information on the set of weapons and equipment of the opposing side for any set. The method of composing and solving matrix games in game theory is used. The problem is modeled as a matrix game with mixed strategies, and the existence of its solution (saddle point) is shown in this setting. To obtain a numerical solution of the problem, the matrix game is reduced to the linear programming problem.

Key words: weapon category, weapon impact index, ability to destroy the weapon, static power, dynamic power, mixed strategy, cost function.

1. INTRODUCTION

Success of any tactical activity is contingent upon its proper organization. This depends, along with the numerous factors, on the one hand, on the total power of weapons and equipment gathered by the enemy in the area of operations on the basis of intelligence data, and on the other hand, on the selection of weapons and equipment intended to destroy them. Determining the number of required weapons and equipment depending on its form, type, operational conditions during the preparation of an offensive battle is explained in a number of scientific and technical literature.

Classical approaches to the order of selection of weapons are reflected in textbooks and in instructions. (e.g. Babayev:2017, Babayev:2016a et al 2016, Popov:2005, Aldo:2013).

Among novel studies, the issue of the optimal choice of resources used to achieve victory was considered in the works of S.Babayev based on information about the weapons and equipment of the enemy. The value of the equipment and weapons used or the number of probable casualties was taken as the optimality criterion (e.g. Combat:2015, Tank:1997, Babayev:2016b).

The experience of recent military conflicts in different parts of the world shows that information on the weapons and equipment of the opposing side is not sufficiently accurate, but it is possible to obtain information on their overall dynamic power on the basis of various indirect estimates (e.g. Onişor:1999, Bowdish: 2013).

Depending on the intended tactical action, the power of the weapon and equipment required to defeat an enemy of known strength is usually considered known. It is clear that the options for organizing forces with such power can be different.

This article studies the rule of optimal organization of forces to achieve a guaranteed victory, regardless of the choice of weapons, when the enemy's dynamic strength is known.

2. BASIC CONCEPTS OF MILITARY OPERATIONS

A number of concepts that characterize the weapons used in military operations (e.g. Mark: 2016) is essential for the purposes of this article and is briefly overviewed below.

The meaning of the weapon category. Depending on the type of operation (offensive or defensive), the concept of the meaning of weapons and equipment related to each category (category of importance) is included. According to their purpose and role in military operations, military weapons and equipment are divided into 10 categories. The value of weapons and military equipment for different categories is given in the form of tables in the military-technical literature. (Aldo:2013, p.2, Popov: 2005, p.43). It should be noted that the numbers given in the tables are relative in nature, depending on the opinion of experts, their mutual ratio may have different values in different sources. (Aldo:2013, p.7, Popov:2005, p.44).

Weapon impact index. Weapons belonging to one category or another differ from each other in terms of their tactical and technical indicators, and a power index is used to show this diversity. (Aldo:2013, p.2, Popov:2005, p.43).

The power index of the weapon is determined depending on its firing capability, maneuverability in military operations, operational capability and application. This indicator is calculated separately for each weapon type and is fixed. The power indexes are calculated by specialists in this field at weapons factories (Aldo:2013, p.6, Popov: 2005, p.44).

Weapon effectiveness index. During military operations, the extent to which weapons and military equipment can destroy enemy forces is characterized by its effectiveness. It is considered that the weapon effectiveness coefficient depends on the nature of the combat work, and the following formula can be used to calculate it. (Aldo: 2013, p.7, Popov: 2005, p.43):

$$E = K \times T \quad (1)$$

Here E - effectiveness index, T - weapon effect index, K - the importance of the category to which the weapon belongs. It should be noted that depending on the nature of the military operation K is different. For example, in the offensive battle of the tank, the effectiveness coefficient is $E = 2$ with the importance of the category K=1, and the effect index $T = 2$.

In a defensive battle when K=3, effect index is $T = 2$ effectiveness coefficient is $E = 6$.

Static strength of a military unit (divison). Static durability characterizes units in terms of available weapons and is determined by the overall rating of all types of weapons available in units. The following formula is used to calculate the static strength of a military unit or division (Aldo:2013, p.8, Popov: 2005, p.45):

$$S = \sum_{j=1,2,...} N_j \cdot E_j \quad (2)$$

Here, S – static strength of unit, j - serial number applied in any order to the various weapons belonging to the combination ($j = 1, 2, ...$), E_j - effectiveness coefficient of j type weapons which determined by the formula (1), N_j – number of j type weapons which belonging to unit.

Dynamic strength of a military unit (division). Military experts recommend to note the specifics of the operation (for the attacking side - the form and type of attack, for the defending side - the level of readiness of the defense, the state of the barriers created) to assess the results of the battle. The indicator calculated taking into account these factors is called dynamic strength. (Aldo:2013, p.19, Popov: 2005, p.43).

The comparative advantage coefficient is used to calculate the dynamic strength. This coefficient shows the degree of superiority of different units (tank, motorized rifle, etc.) over each other and are given in the relevant tables uyğun (table 1) (Aldo:2013, p.19, Popov: 2005, p.43).

Table 1. The comparative advantage coefficient

Units	Tank	Motorized rifle	Small arms
Tank	1.0	1.7	2.0
Motorized rifle weapons	0.6	1.0	1.7
Small arms	0.5	0.6	1.0

Using the comparative advantage coefficient
the dynamic strength of the parties is calculated by formulas (3) - (5):

$D_1 = U_{1,2} \cdot I_1 \cdot S_1, \qquad D_2 = I_2 \cdot S_2,$ (3)

Here, S_1 and S_2 is a static strength of own and opposite sides, respectively, $U_{1,2}$ - coefficient of comparative advantage of one's side in comparison with the opponent's side.

I_1, I_2 and their numbers are determined as follows: - when their side is on the attack

$I_1 = H_1 \cdot H_2, \qquad I_2 = M_1 \cdot M_2,$ (4)

- when their side is defense

$I_1 = M_1 \cdot M_2, \qquad I_2 = H_1 \cdot H_2,$ (5)

Here, H_1 - attack form coefficient, H_2 - attack type coefficient, M_1 - defense readiness coefficient, M_2 – is called obstacle condition coefficient. Their value is given in table 2-5. (Aldo:2013, p.20, Popov: 2005, p.47).

Table 2. Attack forms

Attack form	***Coefficient***
Frontal attack	1.0
Flank attack	2.0
Attack from behind	4.0
Air landing	at the beginning of the battle 0.5
	after 2 hours 4.0
Amphibious warfare	at the beginning of the battle 0.7
	after 2 hours 1.0

Table 3. Attack types

Attack fom	***Coefficient***
Repared attack	1.0
Face-to-face battle	1.2
Sudden attack	1.5

Table 4. Preparedness defense degree

Time taken to prepare the defense before the start of the operation	***Coefficient***
less than 6 hours, unprepared	1.0
from 6 hours to 24 hours	1.2
more than 24 hours	1.4
Long-term, provided with facilities	2.0

Table 5. The barrier system state

State of bariers	***Coefficient***
No barriers	1.0
Weak barriers	1.2
Medium barriers	1.4
Strong bariers	1.6
Too strong barriers	1.8

3. STATEMENT OF THE PROBLEM

When the dynamic strength of the opponent is known, the right choice of weapons must be made to achieve a guaranteed victory. In classical approaches, information about the number of weapons possessed by the opposing side is used to study the organization of tactical activities. For example, selection of weapons and equipment, minimizing their overall cost in (Babayev: 2016b), was carried out under the condition of minimizing possible manpower losses in (Babayev:2016c).

In the presented work, it is considered that only the dynamic strength of the enemy's forces is known. It requires a choice of weapons and equipment in order to achieve a guaranteed victory, regardless of the resources on which the dynamic power of the enemy is formed.

The method used to solve the problem is based on antagonistic game theory. Thus, the study of the organization of tactical activity is carried out with regard to the attacking or defending side. The task is solved by finding a saddle point, which is used in game theory (Wentzel: 2001, p.182).

As is known, models and methods of decision-making in conflict situations are studied in game theory. Considered problem is an antagonistic decision-making problem for two players.

To state the problem, let's give a brief introduction to some game theory concepts. In the considered type of antagonistic game, it is required that the given cost function be maximized by player I and minimized by player II. If the desicion made first by player I and then player II, then the resulting value of the cost function is called maxmin, if the decision is made first by player II and then player I, then the resulting value of the cost function is called ***minmax***. When maxmin equals minmax, this valie is called the price of game and the option that realizes this price is called the saddle point. One class of antagonistic games is called matrix games. The game is implemented by choosing one of the players the rows of the matrix and the other the columns of the matrix. At this time, the meaning of the different options of the parties is determined by the elements of the matrix. If the parties in the game are able to choose only one row and one column, such game is called a pure strategy game. It is well known that pure strategy matrix games do not always have a saddle point. If the parties in the game are able to choose different rows and columns with certain weights, such game is called a mixed strategy game. It is known from game theory that a mixed strategy game always has a saddle point, in other words, there is always a solution for a mixed strategy game. (Wentzel: 2001, p.182).

If the dynamic strength of the opponent is known, so let's express the issue of weapon selection based on game theory terminology. Let's number the types of weapons and equipment that the enemy can use as $j = 1,2, \dots, n,$ and the types of methods our side can use as $i = 1,2, \dots, n,$. Let's denote theability of i -type weapon to destroy the j -type of weapon as $c_{i,j}$. In military affairs quantities of $c_{i,j}$ are considered known (*e.g.* Onishor:1999, Bowdish: 2013), calculated according to the tables above. $c_{i,j}$-s can be represented as the following matrix $\boldsymbol{C}$:

$$\boldsymbol{C} = \begin{pmatrix} c_{1,1} & \cdots & c_{1,n} \\ \cdots & \cdots & \cdots \\ c_{n,1} & \cdots & c_{n,n} \end{pmatrix} \qquad (6)$$

In the following formulas, we will differentiate the quantities related to the parties participating in the military operation, with the superscript index $s = 1,2$ written in. Let $x_i^{(s)}$ denote the number of the s party wants to involve in the operation from the i type of the weapon. Then, the dynamic strength of the parties for the used weapons calculated according to next formulas.

$$\sum_{i=1}^{n} d_i^{(s)} x_i^{(s)} = D^{(s)}, \qquad s = 1.2 \qquad (7)$$

The determinant of the matrix $\boldsymbol{C}$ can be taken non-zero, so that all elements of the matrix are non-zero, and the problem statement remains unchanged if the value of any element is changed slightly. Denote $\|\boldsymbol{C}\| = \det(\boldsymbol{C})$. The matrix(6) can always be written as $\boldsymbol{C} = \|\boldsymbol{C}\| \cdot \boldsymbol{E}$, where

$$\boldsymbol{E} = \begin{pmatrix} e_{1,1} & \cdots & e_{1,n} \\ \cdots & \cdots & \cdots \\ e_{n,1} & \cdots & e_{n,n} \end{pmatrix}.$$

It is clear that the elements of the matrix $\boldsymbol{E}$ are calculated as follows and can be considered known:

$$e_{i,j} = \frac{c_{i,j}}{\|\boldsymbol{C}\|}, i, j = 1,2, \dots, n.$$

Let first side weapon choice is $x_i^{(1)}$, $i = 1,2, \dots,$ and second one weapon choice is $x_j^{(2)}$, $j = 1,2, \dots, n$. Depending on the type of mutual operation carried out by the parties (for example, attack-defense), if we denote the dynamic power conditioned by each weapon as $d_i^{(1)}$, $i = 1,2, \dots, n$ and $d_j^{(2)}$, $j = 1,2, \dots, n$ respectively. For the first side the dynamic power corresponding to the number of weapons $x_i^{(1)}$ is equal to $d_i^{(1)} x_i^{(1)}$, for the second side the dynamic power corresponding to the number of weapons $x_j^{(2)}$ is $d_j^{(2)} x_j^{(2)}$. Then according to the proposed requirements regarding the total dynamic power of parties can be calculated as

$$D^{(1)} = \sum_{i=1}^{n} d_i^{(1)} x_i^{(1)}, \qquad (8)$$

$$D^{(2)} = \sum_{j=1}^{n} d_j^{(2)} x_j^{(2)}. \qquad (9)$$

Here, $D^{(1)}$ and $D^{(2)}$ are known quantities.

We will distribute the dynamic power distribution vector (8) $i = 1,2,\dots,n$ by given types of weapons as $\mathbf{\Lambda} = (\lambda_1, \lambda_2, \dots, \lambda_n)$, and the dynamic power distribution vector (9) by given types of weapons $j = 1,2,\dots,n$ as $\mathbf{M} = (\mu_1, \mu_2, \dots, \mu_n)$:

$$\lambda_i = \frac{d_i^{(1)} x_i^{(1)}}{D^{(1)}}, \qquad (10)$$

$$\mu_j = \frac{d_j^{(2)} x_j^{(2)}}{D^{(2)}}. \qquad (11)$$

From the definitions (10) and (11) it is clear that

$$\sum_{i=1}^{n} \lambda_i = \sum_{i=1}^{n} \mu_i = 1.$$

Then, according to the distribution vector $\mathbf{\Lambda}$, the force created by all weapons of the first side against one unit j weapon of the second side will be calculated as

$$\sum_{i=1}^{n} e_{i,j} \lambda_i. \qquad (12)$$

In order for the power calculated according to (12) to correspond to the dynamic power of the corresponding (j -th type) weapon of the second side, we distribute it between these means along the vector $\mathbf{M}$. Then, if we calculate the dynamic power directed by the i type of weapon against j type of weapon of the second side so that the first side does not lose the game, then we get the expression $e_{i,j} \lambda_i \mu_i$.

It is easy to see that this expression is calculated as the elements of the following matrix product:

$$\boldsymbol{A} = \mathbf{M} \cdot \mathbf{E} \cdot \mathbf{\Lambda}.$$

The sum of the elements of matrix $\boldsymbol{A}$ can be taken as the cost of the game:

$$J \equiv \sum_{j=1}^{n} \sum_{i=1}^{n} e_{i,j} \lambda_i \mu_j. \qquad (13)$$

Let take the following notation:

$$x^{(1)} = \left(x_1^{(1)}, \dots, x_i^{(1)}, \dots, x_n^{(1)}\right),$$

$$x^{(2)} = \left(x_1^{(2)}, \dots, x_j^{(2)}, \dots, x_n^{(2)}\right).$$

If we express the quantities λ_i and μ_j from formulas (10) and (11) by $x_i^{(1)}$ and $x_j^{(2)}$, and the elements of the matrix $\mathbf{E}$ express by the elements of the matrix $\boldsymbol{C}$, the function (13) will be written as follows:

$$J\left(x^{(1)}, x^{(2)}\right) \equiv \frac{1}{D^{(1)} D^{(2)} \|\boldsymbol{C}\|} \times$$

$$\times \sum_{j=1}^{n} \sum_{i=1}^{m} c_{i,j} d_i^{(1)} d_j^{(2)} x_i^{(1)} x_j^{(2)} \qquad (14)$$

The function $J\left(x^{(1)}, x^{(2)}\right)$ is linear in both its arguments. From game theory, it is known that function (14) has a saddle point, i.e., $\min_{x^{(1)}}\max_{x^{(2)}} J(x^{(1)},x^{(2)}) = \max_{x^{(2)}}\min_{x^{(1)}} J(x^{(1)},x^{(2)})$.

Thus, the question of choosing a weapon to achieve a guaranteed victory can be formulated in the language of the theory of matrix games as follows:

- The value of the function $J(x^{(1)},x^{(2)})$ is maximized by the first player (our side, by choosing the parameter $x^{(1)}$), and the second player (the opposite side, by choosing the parameter $x^{(2)}$) minimizes it.

4. REDUCING THE TASK TO THE LINEAR PROGRAMMING PROBLEM

Denote the cost of the game by U:

$$U \equiv \max_{x^{(2)}}\min_{x^{(1)}} J(x^{(1)},x^{(2)}). \qquad (15)$$

By considering the condition for each $j = 1,\dots,n$, $\mu_j \geq 0$ and $\mu_1 + \dots + \mu_n = 1$, so from the obvious form of the function $J(x^{(1)},x^{(2)})$, we can get next equality

$$\sum_{i=1}^{n}\left(\frac{1}{\|C\|}\sum_{j=1}^{n} c_{i,j}\right)\frac{d_i^{(1)}x_i^{(1)}}{D^{(1)}} \leq U. \qquad (16)$$

If we denote

$$y_i = \frac{d_i^{(1)}x_i^{(1)}}{D^{(1)}U} \qquad (17)$$

the inequalities (16) can be written as follows:

$$\sum_{i=1}^{n} H_i y_i \leq 1,\ \ j = 1,2,\dots,n, \qquad (18)$$

here $H_i = \frac{1}{\|C\|}\sum_{j=1}^{n} c_{i,j}$. According to (17), it is clear that

$$y_i \geq 0, \qquad i = 1,2,\dots,n. \qquad (19)$$

From the other hand, according to equations of (8) and (17)

$$\sum_{i=1}^{n} y_i = \frac{1}{U}\sum_{i=1}^{n}\frac{d_i^{(1)}x_i^{(1)}}{D^{(1)}} = \frac{1}{U}.$$

From the equivalence of the requirement $U \to \min$ to the requirement $\frac{1}{U} \to \max$, it is obtained next criteria from the last equation

$$\sum_{i=1}^{n} y_i \to \max. \qquad (20)$$

Thus, the (15)-(17) matrix game of mixed strategies reduces to a linear programming problem (18)-(20) and can be solved, for example, using the simplex method (Wentzel: 2001, p.52).

5. CONCLUSION

One of the characteristics of tactical groups in combat operations is dynamic power. Dynamic power is calculated on the basis of the combat conditions, the planned combat, and the weapons and equipment used. Thus, the assessment of dynamic power depends on the set of weapons and equipment.

The proposed approach can be used to select weapons during the tactical planning phase when there is an overall estimate of the enemy forces. This requires that the coefficients of the ability of each type of weapon to destroy other types of weapons be known. But the diversity of the fleet of military weapons and equipment of different countries and their new modifications in recent years require that appropriate research be conducted to calculate these coefficients. It is assumed that the coefficients expressing the ability of each weapon to destroy other weapons are known.

In the paper, the problem of selecting weapons and equipment to achieve a guaranteed victory in tactical actions, when the dynamic power of the enemy is known, is simulated as a mixed strategy matrix game. It is shown that the resulting game theory problem can be reduced to a linear programming problem and solved by numerical methods. The selection of weapons and equipment can be considered and solved as a mixed strategy matrix game.

It can be noted that in other conflict situations, if it is possible to calculate the interaction of relevant factors, it is possible to use game theory to calculate a guaranteed result.

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